

Class 16 - § 4.6 Graphs of Rational Functions

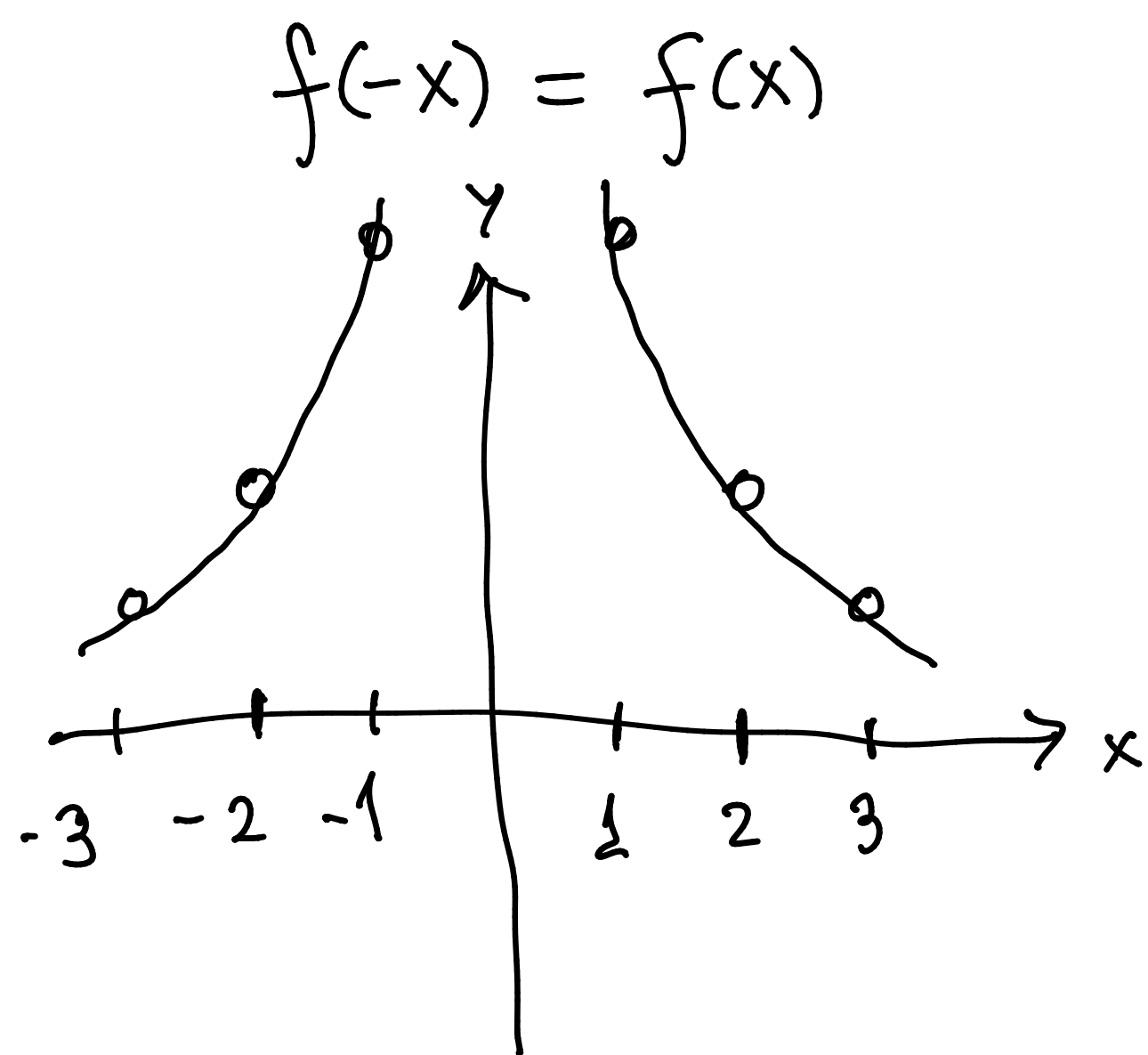
Review

• Rational function = $\frac{\text{Polynomial}}{\text{Polynomial}}$

$$\frac{x^2 - 4}{2} = \frac{1}{2}x^2 - 2 \quad \text{Not rational}$$

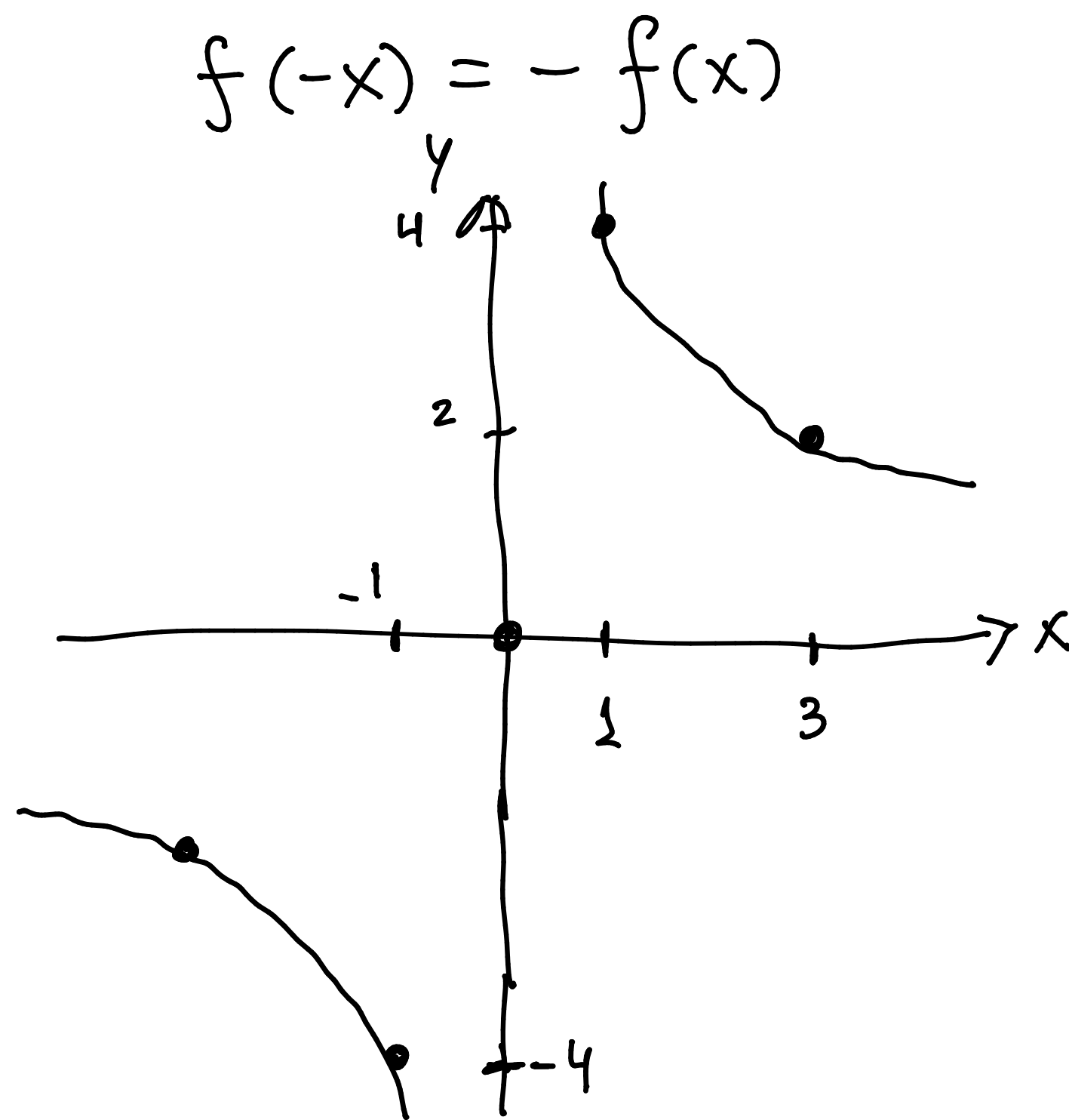
$$\frac{x^2 - 4x + 4}{2x - 1} \quad \text{is rational}$$

• Even vs. Odd functions:



Mirror reflection

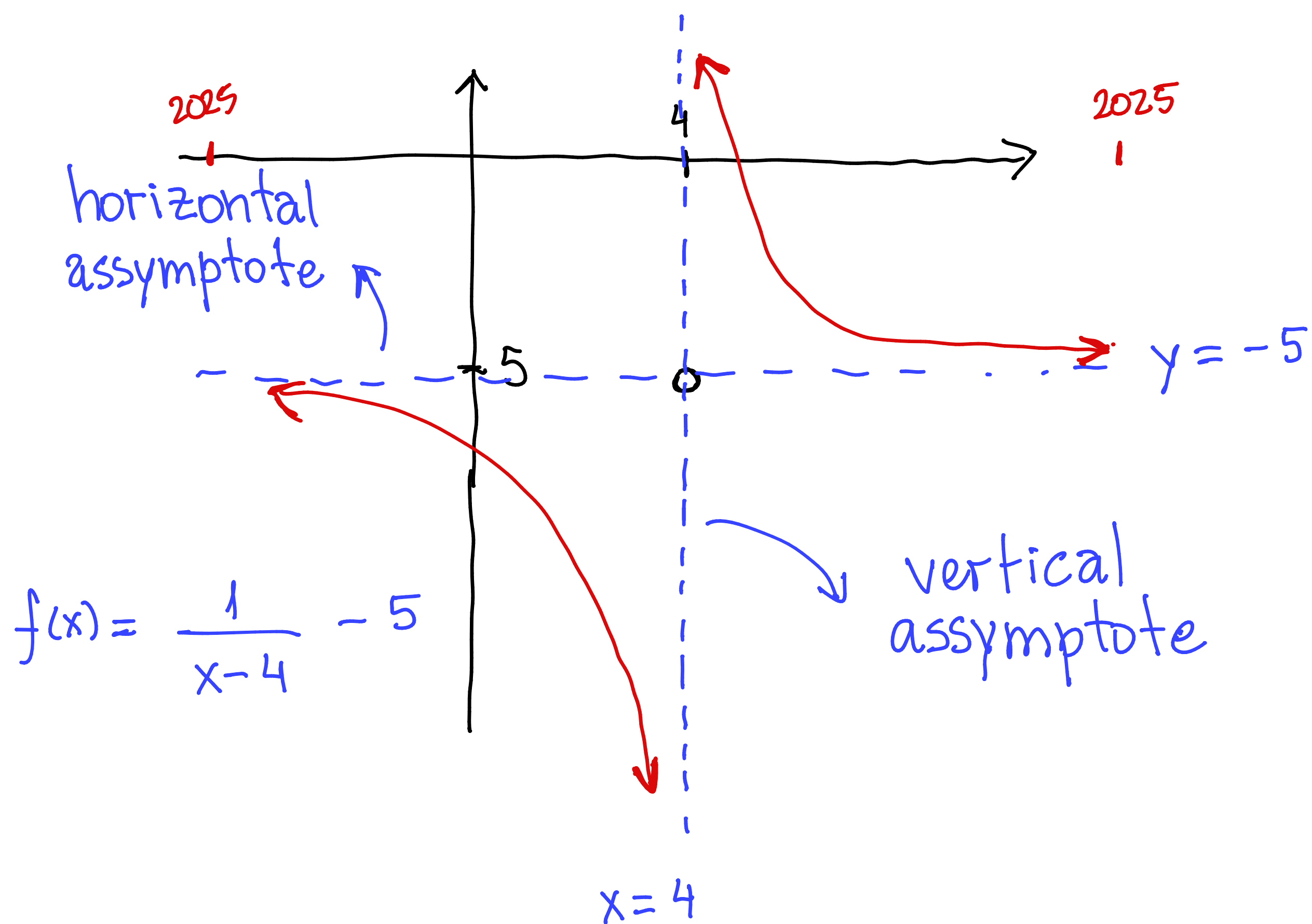
w.r.t. y-axis



→ Transformation of the Reciprocal function $\frac{1}{x}$

Ex: $f(x) = \frac{1}{x-4} - 5$

shift 4 units → shift 5 units ↓



$$f(3.99) = \frac{1}{-0.01} - 5 = -105$$

$$f(4.01) = \frac{1}{0.01} - 5 = 95$$

$$f(2025) = \frac{1}{2021} - 5 \approx -4.99906$$

$$f(-2025) = \frac{1}{-2029} - 5 \approx -5.0005...$$

→ Simplify rational expressions & Find restricted values

$$\text{Ex 1: } \frac{5x^2 - 21x - 20}{5x^2 - 11x - 12} = \frac{5x^2 - 25x + 4x - 20}{5x^2 - 15x + 4x - 12} = \frac{5x(x-5) + 4(x-5)}{5x(x-3) + 4(x-3)} = \frac{(x-5)(5x+4)}{(x-3)(5x+4)}$$

$$\frac{x-5}{x-3}$$

Restricted values: #'s that make denom. equal to zero.

$$5x^2 - 11x - 12 = 0 \Leftrightarrow \underbrace{(x-3)}_A \underbrace{(5x+4)}_B = 0$$

$$x=3 \quad x=-\frac{4}{5}$$

→ Finding intercepts of an equation or function:

x-intercept: set $y=0$ or $f(x)=0$

y-intercept: set $x=0$ or $f(0)$.

$$\text{Ex 1: } f(x) = \frac{3x^2 - 4x - 7}{4x-1} = \frac{(x+1)(3x-7)}{4x-1} \quad f(0) = \frac{1 \cdot (-7)}{-1} = 7$$

$$0 = \frac{(x+1)(3x+7)}{4x-1} \Rightarrow 0 = (x+1)(3x-7) \Leftrightarrow x = -1, x = \frac{7}{3}$$

$$\frac{3x^2 - 7x + 3x - 7}{x(3x-7) + 1(3x-7)}$$

$$(x+1)(3x-7)$$

§ 4.6 Graph of Rational Functions

Three very important things:

- 1) Vertical asymptotes $\rightarrow$ den. = 0 that does not simplify
- 2) Horizontal asymptotes $\rightarrow$ compare the degree of polynomials
- 3) Removable discontinuities $\rightarrow$ den. = 0 that does simplify

$$\text{Ex 1: } \frac{5x^2 - 21x - 20}{5x^2 - 11x - 12} = \frac{(x-5)(\cancel{5x+4})}{(x-3)(\cancel{5x+4})} = \frac{x-5}{x-3}$$

$y = \frac{5}{5} = 1$ is a
horiz. asymptote

$$\text{den.} = 0 \Rightarrow x = 3 \text{ \& } x = -\frac{4}{5} \text{ (Restricted Values)}$$

Vertical
asymptote

Removable
Discontinuity

$$x = -\frac{4}{5}: \frac{-\frac{4}{5} - 5}{-\frac{4}{5} - 3} = \frac{29}{19}$$

$(-\frac{4}{5}, \frac{29}{19})$

$$5x + 4 = 0$$

$$5x = -4$$

$$x = -\frac{4}{5}$$

- * 2) $\left\{ \begin{array}{l} \text{deg. of numerator} > \text{deg. of denominator} \Rightarrow \text{NO horizontal asymp.} \\ \text{deg. of numerator} < \text{deg. of denominator} \Rightarrow \text{asymptote } y = 0 \\ \text{deg. of numerator} = \text{deg. of denominator} \Rightarrow \text{asymptote } y = \frac{LC}{LC} \end{array} \right.$

Graphing Rational functions:

1) Domain

2) Remov. Discont.

3) Symmetries

4) y-intercepts

5) x-intercepts

6) vertical asymp.

7) horiz. asymp.

8) Test values.

9) Graph

Ex: $f(x) = \frac{5x^2 - 21x - 20}{5x^2 - 11x - 12}$.

0) Simplify $\rightarrow f(x) = \frac{(x-5)(5x+4)}{(x-3)(5x+4)} = \frac{x-5}{x-3}$.

1) Domain $\rightarrow$ all #'s except restricted values: $(-\infty, -\frac{4}{5}) \cup (-\frac{4}{5}, 3) \cup (3, \infty)$
 $(3 \text{ \& } -\frac{4}{5})$

2) Remov. Discont. $\rightarrow x = -\frac{4}{5}, y = \frac{29}{19} \Rightarrow (-\frac{4}{5}, \frac{29}{19})$ -0.8

3) Symmetries $\rightarrow$ No symmetries. (0, 1.66...)

4) y-intercepts $\rightarrow f(0) = \frac{0-5}{0-3} \rightarrow f(0) = \frac{5}{3} \cdot (0, \frac{5}{3})$ (0, 1.66...)

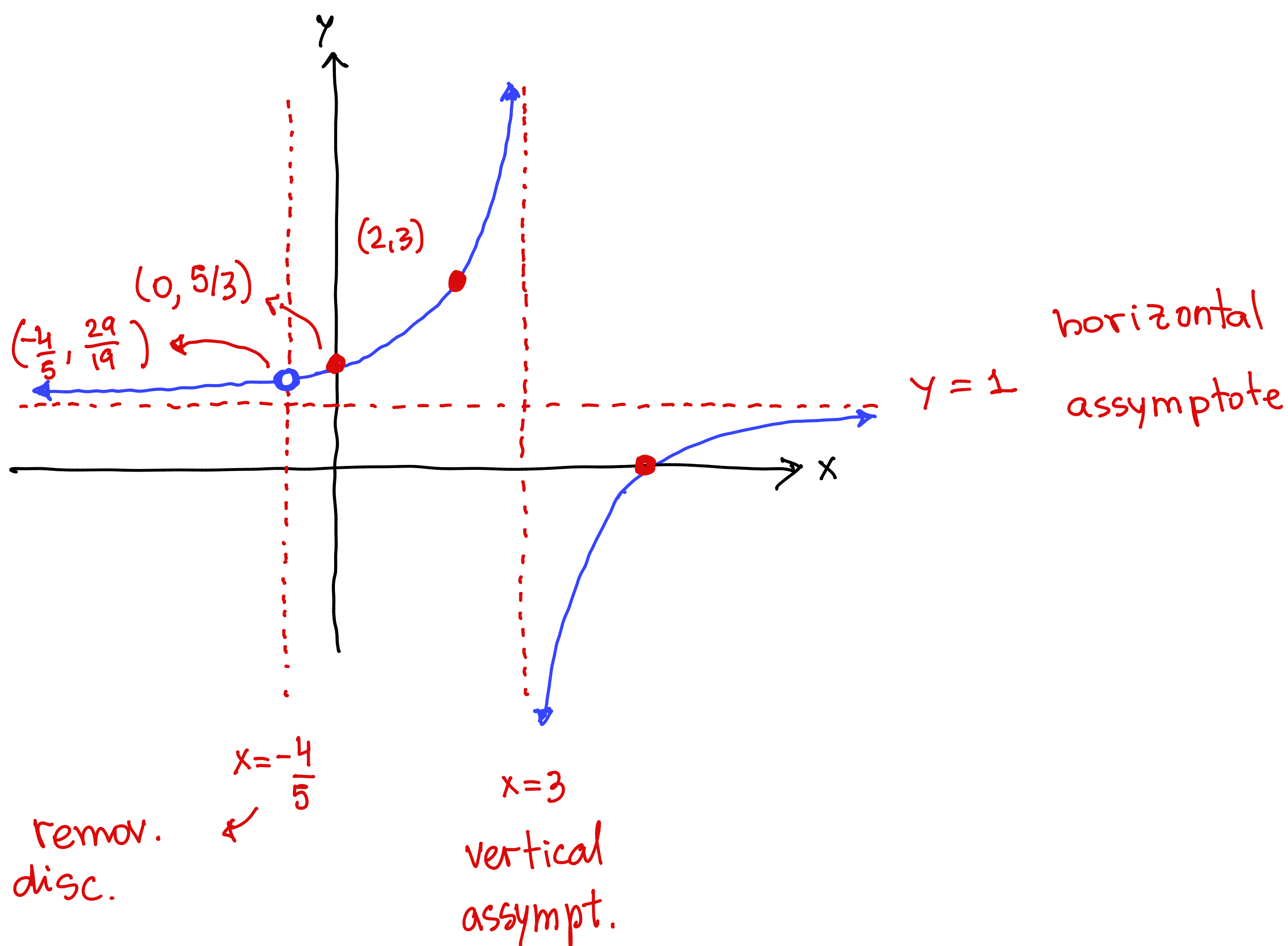
5) x-intercepts $\rightarrow f(x) = 0 \rightarrow 0 = \frac{x-5}{x-3} \rightarrow 0 = x-5 \rightarrow x=5. (5, 0)$.

6) vertical assymp. $\rightarrow \boxed{x=3}$

7) horiz. assymp. $\rightarrow \frac{\text{deg. 2}}{\text{deg. 2}} \rightarrow \frac{LC}{LC} \rightarrow y = \frac{5}{5} = 1 \quad \boxed{y=1}$

8) Test values. $f(2) = \frac{2-5}{2-3} = \frac{-3}{-1} = 3$

9) Graph



Ex 2: Graph the function $f(x) = \frac{5x^2 - 10x - 240}{x^2 - 14x + 48}$

0) Simplify: $\frac{5(x^2 - 2x - 48)}{x^2 - 14x + 48} = \frac{5(x+6)(x-8)}{(x-6)(x-8)} = \frac{5(x+6)}{x-6}$.

1) Domain: all #'s except restricted values: $(-\infty, 6) \cup (6, 8) \cup (8, \infty)$

denominator = 0 $\Leftrightarrow x^2 - 14x + 48 = 0 \Leftrightarrow (x-6)(x-8) = 0 \Leftrightarrow x = 6, 8$

2) Remov. Disc: den. = 0 & cancel when simplifies: $(8, 35)$

3) Symmetries: No symmetries.

4) y-intercept: $x=0: y=-5$ or $f(0)=-5 \rightarrow (0, -5)$

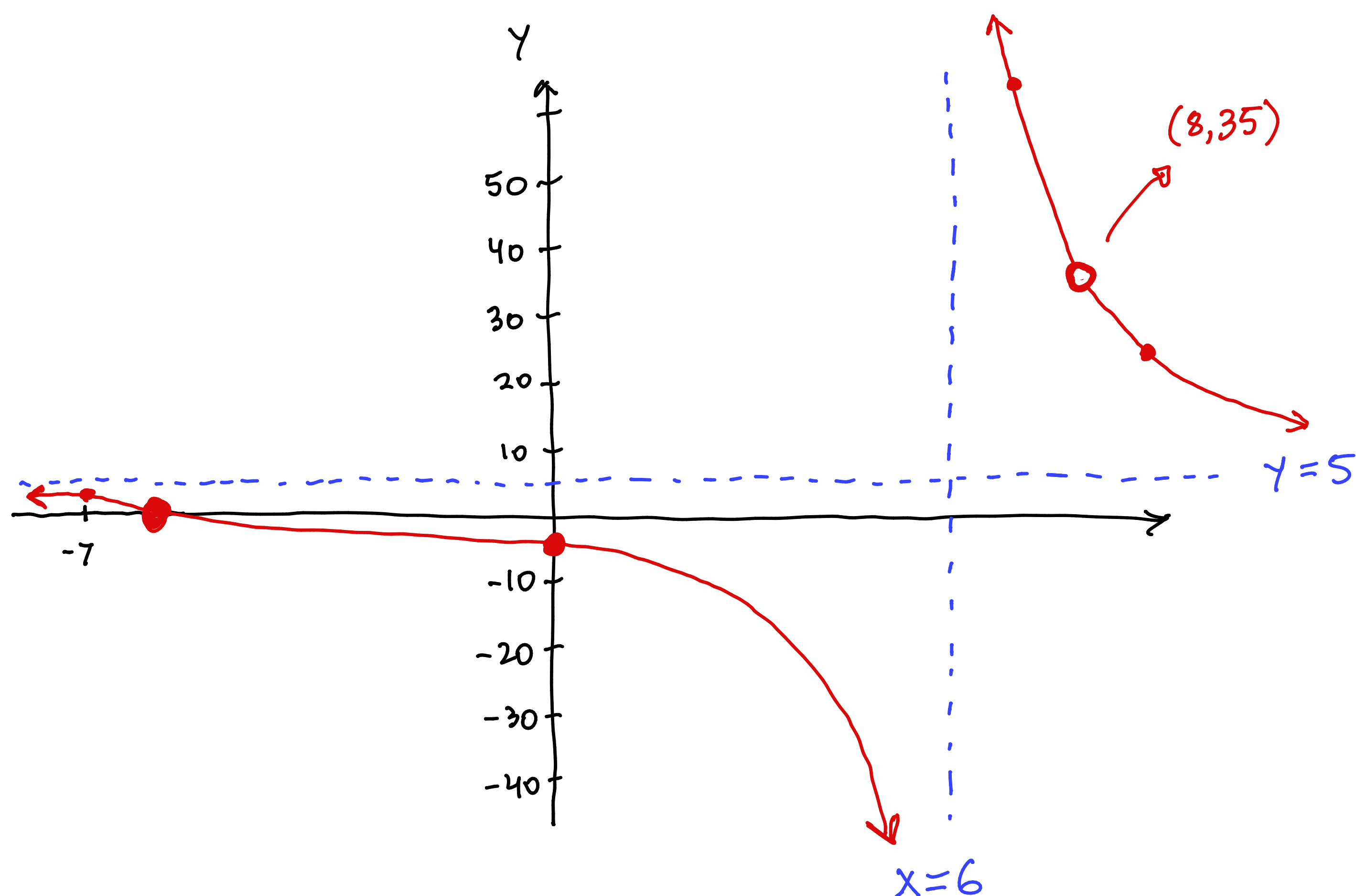
5) x-intercept(s): $y=0$ or $f(x)=0 \rightarrow 5(x+6)=0 \rightarrow x=-6. (-6, 0)$

6) Vert. asymptotes: $x=6$

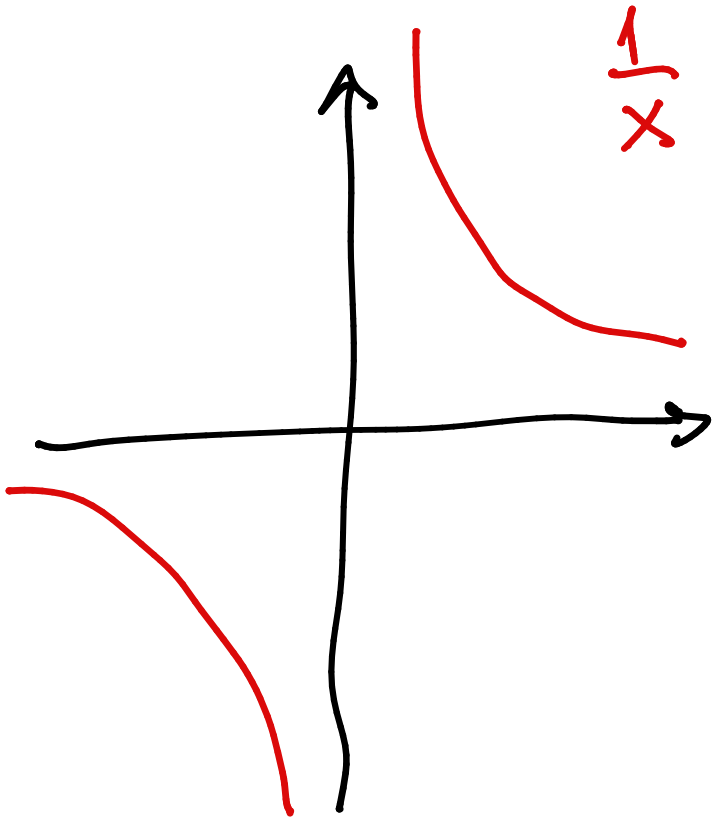
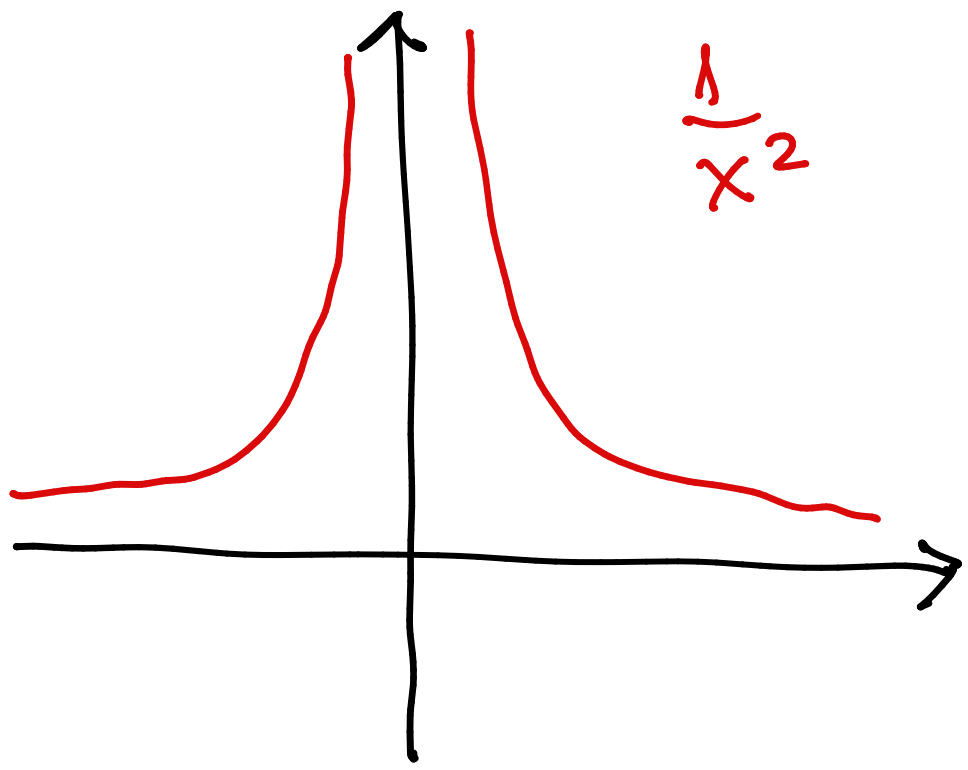
7) horiz. asymptotes: $y = \frac{5}{1} \rightarrow y = 5$

8) Test Values: $f(-7) = \frac{5}{13}$, $f(9) = 25$, $f(7) = 65$

9) Sketch the graph:



Special Graphs: $\frac{1}{x}$ & $\frac{1}{x^2}$

Graph		
Domain	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
Range	$(-\infty, 0) \cup (0, \infty)$	$(0, \infty)$
Intercepts	NO	NO
Assymp.	$x=0$ & $y=0$	$x=0$ & $y=0$
Symmetry	$f(-x) = -f(x)$ ODD	$f(-x) = f(x)$ EVEN